The Three R's of Mathematics

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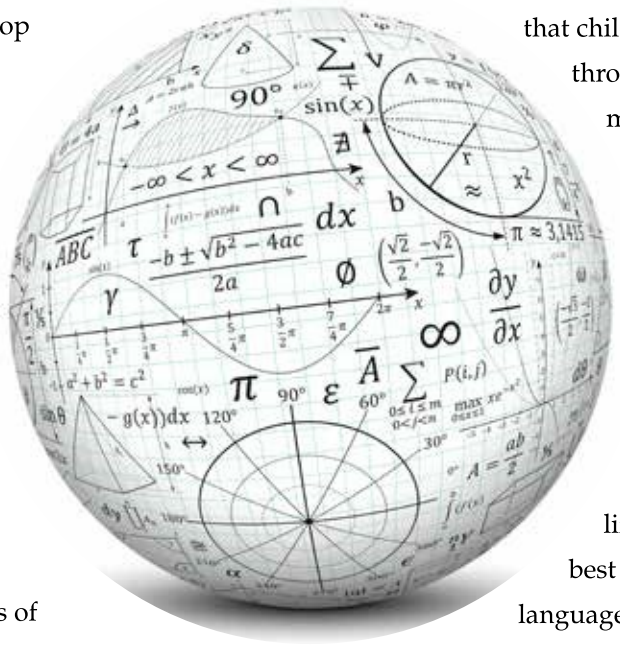
Success in mathematics requires a variety of skills, all of which are perfectly situated within classical Christian schools. Classical Christian educators can use God's Word to help students develop these skills. Recognizing the good information from the bad is a key objective sought by classical educators. The ability to see truth in a world full of untruths is imperative. The same skill applies to mathematics: Students must be able to recognize relationships in mathematics to be able to know how to proceed. This recognition, the first R, will help students get started on one of the toughest parts of mathematics at any age, problem-solving. The second R, retrieval, is a basic tenet of classical education: Students must memorize their basic facts and be able to retrieve the facts quickly. Finally, the third R, resolve, can be taught both biblically and through developing students' mindsets. Classical Christian educators have at their disposal biblical truths in developing students' resolve.



RECOGNITION

Developing students who have good number sense is critical in mathematics. Nguyen et al. discovered that early numeracy ability in preschool is a strong predictor of fifth-grade mathematics achievement scores (2016). What is number sense? It is a group of skills that allows people to work with numbers. Witzel, Ferguson and Mink discuss five components of number sense: magnitude comparisons, strategic counting, retrieval of basic arithmetic facts, word problems and numerical recognition (2012). The authors go on to discuss three methods for improving number sense.

First, the authors support constructivist claims that children construct their knowledge through manipulating concrete materials. Second, they discuss how proficiency of skill should not just include algorithms, but also the meaning behind the algorithms. Finally, the third key element of the theory, the importance of making language connections, is offered to integrate math to everyday life. The first R, recognition, is best accomplished through these language connections. In her article on this idea of language in mathematics, Susperreguy emphasizes the importance of math talk, specifically the use of language comparisons. The use of math talk that includes cardinality and counting is ubiquitous in homes. What is missing, according to Susperreguy, is the use of comparisons: more than, less than, parts and wholes (2016). It is the recognition of parts and wholes in problems that unlocks mathematical understanding. Knowing that two parts are given in a problem allows the solver to add, no matter what the numerals are that are being added. Knowing that a whole and a part are given allows the solver to subtract. Taking time to recognize the information in the problem is key. This is best understood in the context it is required: problem-solving.

Nicholas needed to distribute 5 ¼ bags of grass seed on a lawn. He distributed 3 ½ bags in the morning. What is the total amount of seed he still needs to distribute before running out?

These rational adjectives (5 ¼ and 3 ½) can sometimes cause angst for students, and students then struggle with knowing which operation to choose. What if, on the other hand, the problem had no fractions in it?

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The problem becomes much easier for upper elementary students and they can immediately recognize that a whole and a part are known and that they need to subtract. By using the language of parts and wholes, students recognize the relationship and are on their way to solving. Knowing the operation required to solve problems eliminates one of the most common errors: choosing the wrong operation (Ferrucci, Yeap, & Carter, 2003). The same idea applies to multiplication and division, the only difference being that the parts are equal parts and that if you have equal parts and know the number of parts, you multiply to determine the product.

When we encourage students to seek truth in the relationships and carefully work through problems, we encourage the biblical virtue of carefulness. Phillip Dow writes in his book *Virtuous Minds* that, "Those who are intellectually careful earnestly want to know the truth; thus, they are reasonable and consistently careful that they do not overlook important details and habitually avoid hasty conclusions based on limited evidence." When we teach students to take time to discover parts and wholes in mathematical problems, we are teaching this carefulness. And finally, from John 8:32, "And you will know the truth, and the truth will set you free." God's truth illuminates the need for seeking truth.

RETRIEVAL

Retrieval of basic mathematics facts is a hot topic in education. Classical educators, however, have consistently held students responsible for memorizing basic facts. The importance of quick retrieval of basic facts cannot be overemphasized. A study by Calderon-Tena and Carerino in the *Journal of Science and Mathematics Education* in 2016 supports this return to holding students accountable for memorizing their basic facts – something classical educators never left. The researchers found that long-term retrieval skills became a better predictor of both mathematics calculation and mathematics problem-solving as age and grade increased.

The time that is devoted to fact retrieval tends to focus most on the initial counting stages and on the ubiquitous practice of timed tests. How to get effective practice at that middle stage will be the focus of this section, and brain-based research will help explain why it is important. In his 2014 book *The Confident Student*, Kanar discusses the three stages of memory: sensory memory, short-term memory and long-term memory. Sensory memory is the memory that takes in information. What a person sees, hears and touches all are taken in and sensed by the brain. If what the brain senses is attended to and processed, then it makes it into short-term memory. Short-term memory manipulates and processes information for about 30 seconds. Finally, if the information is rich enough and engaging enough, the information gets transferred into long-term memory. How does knowing this information help with basic fact retrieval? Simply put, attention matters. Students typically are first taught basic facts through a progression similar to the following: counting, adding zero, doubles, doubles +1, combinations of ten, make ten, doubles +2, +9, +4 in addition, then using addition facts to help retrieve subtraction facts (Purpura, Baroody, Eiland, & Reid, 2016). When they are taught these strategies, such as doubles plus one, teachers use effective manipulatives and visuals to first teach the meaning behind the basic facts. This follows cognitive learning theory first introduced by Jean Piaget and further developed by Jerome Bruner. Bruner significantly added to learning theory by stating that children first need to use concrete manipulatives to learn concepts, then transition into pictures of the objects,

and finally transfer to abstract numerals to represent the number of objects. In math fact retrieval, they may count eight blocks, then add two blocks to work in the concrete stage. Next they might use a ten-frame to show pictures of blocks and visualize that 8 and 2 always make 10. Finally, they will write the equation 8 + 2 = 10 and work with numerals. This type of practice is in every mathematics curriculum in the United States, including those used by classical Christian schools. This is as it should be, for students who have good number sense and practice with rich strategies are more successful at transferring the information into long-term memory (Purpura, Baroody, Eiland, & Reid, 2016).

The question should follow then, why we have so many students who struggle with their retrieval of basic facts? The answer lies in what comes next in schools around the country. Students who initially practice retrieving their facts by spending time counting to retrieve them, such as 8 + 2 = 9, then 10, do not experience the same level of richness as students who associated their facts with known facts. Utilizing what is known in memory to learn unknown information is key to all of learning, but especially to basic fact retrieval. Students must be fair-minded enough to try new methods for retrieving

facts. Dow speaks of the importance of fair-mindedness as well in developing students who have virtuous minds (2013). Fair-mindedness in mathematics is crucial to understanding the subject. Fair-mindedness comes into play in problem-solving, understanding relationships, and yes, in basic fact retrieval. Students who are retrieving their facts by counting as fast as they can should learn new retrieval routes, but in order to do so they must be fair-minded.

Classical Christian schools traditionally emphasize the importance of basic fact retrieval, and they should. I am *not* saying that basic fact retrieval is time wasted. On the contrary, research demonstrates that it is time well spent. What comes next, however, in many schools is the use of timed tests to retrieve basic facts *before* students are ready to be timed. Much research has shown that the overemphasis on timed tests at too early of an age results in math anxiety, something we all want to avoid for our students (Boaler, 2016).

Why not allow more practice for basic fact retrieval within the associative, strategic stage? This is no small task, and I do not mean to trivialize it. Most educators do not know what this looks like. What I am calling for is a change in both curriculum and instructional practices that still allow for accountability, a key component of classical Christian education. Students who are struggling with their fact retrieval do not need more timed tests or more manipulatives. Instead, they need more time associating, or deriving their facts. At our school, The Geneva School of Boerne, students are doing just that. If they show signs of counting or skip-counting while trying to retrieve their addition, subtraction, multiplication or division facts, they are given the tools to help them practice more in the deriving stage. We still require them to spend time retrieving their facts, and we hold them responsible for memorizing those facts. However, using standard flash cards can be just as detrimental to developing math anxiety as timed tests if pressure is placed on students to retrieve them quickly. Rather they should spend time altering their retrieval by associating the unknown fact to known facts. Students need rich practice to transfer information from short-term memory to long-term memory, as Kanar suggests (2014). They also must be fair-minded enough to try new methods to retrieve their facts if they have continuously built the counting pathway in their brain. The second R, retrieval of basic facts, is a key tool that students must possess.

RESOLVE

Finally, the third R, resolve, must be considered as an important characteristic for students to develop. Students who think they can solve math problems are the most successful. Self-efficacy, or beliefs about one's abilities to accomplish goals, can influence activities people participate in (or not), the amount of effort they give to tasks and the persistence of effort and level of achievement reached (Boaler, 2016; Cerit, 2013). Self-efficacy is an area of study that needs to be further investigated in all teacher research studies, but specifically in the content area of mathematics.

Additional research on self-efficacy has been conducted recently by Carol Dweck (2006), who clearly shows the importance of students' mindsets in her book, *Mindset*, by elucidating the difference between students who have a fixed mindset and those with a growth mindset. Those with fixed mindsets believe that they either have a talent, or do not. Those with growth mindsets, on the other hand, believe that if they work hard enough they can learn anything. Boaler has connected mindset research from Dweck to the area of mathematics in her book *Mathematical Mindsets* (2016). Students who have growth mindsets score higher on mathematics achievement tests. Teachers, according to Boaler, can encourage a growth mindset in their students in several ways. For example, the praise that teachers direct towards students is extremely influential. Praise suggesting a student is smart furthers the fixed

mindset, whereas praise suggesting the student has worked hard furthers a growth mindset.

Classical Christian educators, however, have the best tool available to help develop students' mindsets: God's Word. We can first give examples of grit from the Bible. Moses took a long time to reach the promised land and faced great strife. Yet he persevered. We also know from 1 Peter 1:3-5 that we have a promise of hope and that this promise is not wishful thinking, but rather confidence in God's faithfulness. A second way to inspire grit is to remind students of times when they were successful in the past. If you develop a relationship with students and know their past success stories, you will be better equipped to help them through challenges they encounter in the future. The third method for mindset development is to model it yourself as a leader. Students look up to their leaders who have grit and are honest about their struggles. We know that one way students establish their own self-efficacy is by watching it modeled by their peers. Tenacity, or resolve, is a virtuous trait that can be developed by reminding students that hard work pays off. Resolve, a virtuous trait, is worthy of being titled the third R in mathematics.

CONCLUSION

The three R's in mathematics – recognition of relationships, retrieval of basic facts and resolve to work through difficult problems – can be developed by parents, teachers, coaches and mentors. Students need to be surrounded by people who show that they care and take time to help students develop these traits. The Christian virtues of carefulness, fair-mindedness and tenacity can help students develop the three Rs, which, in turn, will help them succeed in their mastery of mathematics.

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